Based on K. H. Rosen: Discrete Mathematics and its Applications.

Lecture 8: Operations with sets. Section 2.2

1 Operations with sets

Definition 1. Let A and B be sets. The **union** of the sets A and B, denoted by $A \cup B$, is the set that contains those elements that are either in A or in B, or in both.

$$A \cup B = \{x \,|\, x \in A \vee x \in B\}.$$

Definition 2. Let A and B be sets. The **intersection** of the sets A and B, denoted by $A \cap B$, is the set that contains those elements that are in both A and B

$$A \cap B = \{x \,|\, x \in A \wedge x \in B\}.$$

Definition 3. Two sets are called **disjoint** if their intersection is the empty set.

Definition 4. Let A and B be sets. The **difference** of A and B, denoted by $A - B$, is the set containing those elements that are in A but not in B. The difference of A and B is also called the complement of B with respect to A. The difference is also denoted by $A \setminus B$.

Definition 5. Let U be the universal set. The **complement** of the set A, denoted by $\bar{A}$ or A', is the complement of A with respect to U. Therefore, the complement of the set A is $\bar{A} = U - A$ or

$$\bar{A} = A' = \{x \in U \,|\, x \notin A\}.$$

Set identities:

1. (Identity laws)
   (a) $A \cap U = A$.
   (b) $A \cup \emptyset = A$.
2. (Domination laws)
   (a) $A \cup U = U$.
   (b) $A \cap \emptyset = \emptyset$.
3. (Idempotent laws)
   (a) $A \cup A = A$.
   (b) $A \cap A = A$.

4. (Complementation law) $\bar{\bar{A}} = A$.
5. (Commutative laws)
   (a) $A \cup B = B \cup A$.
   (b) $A \cap B = B \cap A$.
6. (Associative laws)
   (a) $A \cup (B \cup C) = (A \cup B) \cup C$.
   (b) $A \cap (B \cap C) = (A \cap B) \cap C$.
7. (Distributive laws)
   (a) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.
   (b) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.
8. (De Morgan's laws)
   (a) $\overline{A \cap B} = \bar{A} \cup \bar{B}$.
   (b) $\overline{A \cup B} = \bar{A} \cap \bar{B}$.
9. (Absorption laws)
   (a) $A \cup (A \cap B) = A$.
   (b) $A \cap (A \cup B) = A$.
10. (Complement laws)
   (a) $A \cup \bar{A} = U$.
   (b) $A \cap \bar{A} = \emptyset$.

We can express the union $A \cup B$ as union of three disjoint sets when we write

$$A \cup B = (A - B) \cup (A \cap B) \cup (B - A).$$

Definition 6. The union of a collection of sets is the set that contains those elements that are members of at least one set in the collection. The intersection of a collection of sets is the set that contains those elements that are members of all the sets in the collection. If we take for example our sets as $A_1, A_2, \ldots, A_n$ then

$$\bigcup_{i=1}^{n} A_i = A_1 \cup A_2 \cup \cdots \cup A_n \qquad \bigcap_{i=1}^{n} A_i = A_1 \cap A_2 \cap \cdots \cap A_n.$$

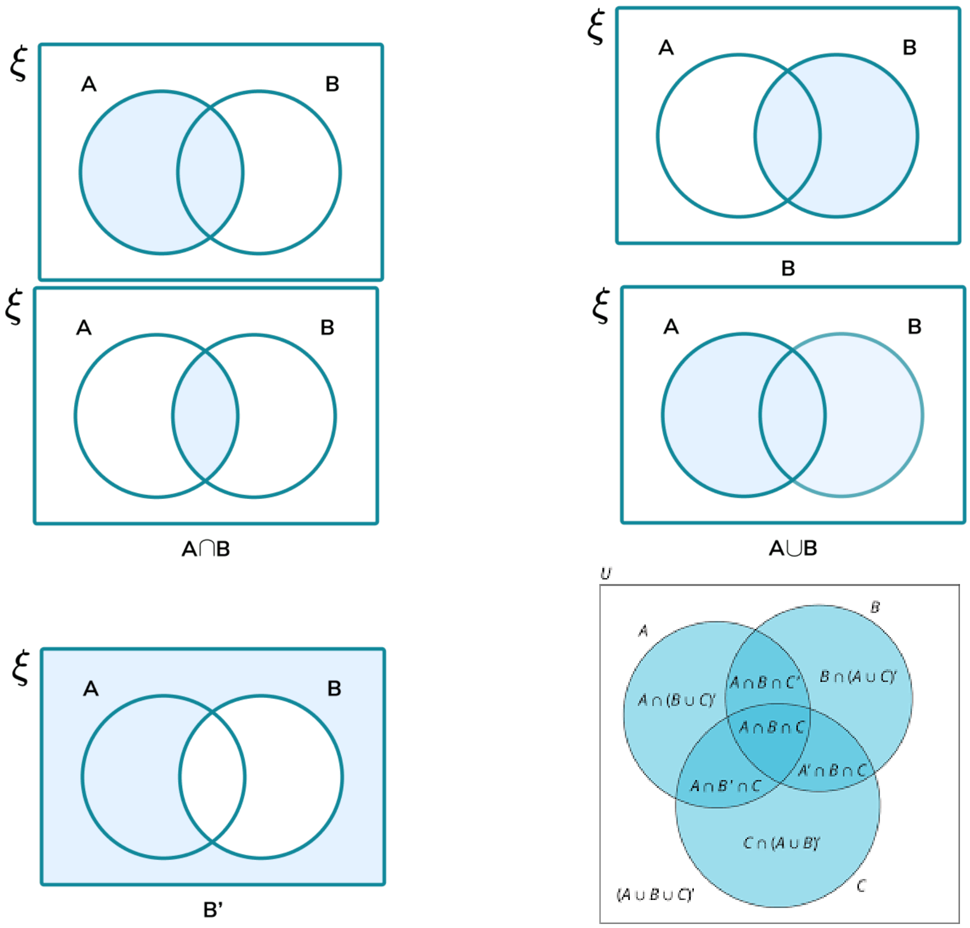

Figure 1: Venn diagrams for the intersection, union and complements.

1.1 Computer representation of sets

We will present a method for storing elements using an arbitrary ordering of the elements of the universal set. This method of representing sets makes computing combinations of sets easy. Assume that the universal set U is finite. First, specify an arbitrary ordering of the elements of U, for instance $a_1, a_2, \ldots, a_n$. Represent a subset A of U with the bit string of length n, where the ith bit in this string is 1 if ai belongs to A and is 0 if ai does not belong to A.

Example 7. In $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ the numbers divisible by three are represented by the string of digits 0010010010.